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Numerical Solution of Nonlinear Variational Problems with Moving Boundary Conditions by Using Chebyshev Wavelets*

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Abstract

We use Chebyshev wavelets on the interval $[0,1)$ *to solve the nonlinear variational problems with moving boundary conditions.An operational matrix of integration is introduced and utilized to reduce the variational problems to the solution set of algebraic equations .Illustrative examples are included to demonstrate the validity and applicability of the technique.*

Keywords: Chebyshev Wavelets, Operational Matrix, Variational Problem

1. Introduction

Special attention has been given to applications of orthogonal functions , such as Walsh functions[6], block-pulse functions[1,10], Fourier series[14], Laguerre polynomials[9,11], Legendre polynomials[5], Chebyshev polynomials[7,12].
There are three classes of sets of orthogonal functions which are widely used.
The first includes sets of piecwise constant basis functions (e.g. Walsh , block-pulse, etc .). The second consists of sets of orthogonal polynomials (e.g. Laguerre, Legendre, Chebyshev, etc.). The third is the widly used sets of sine-cosine functions in Fourier series .While orthogonal polynomials and sine-cosine

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functions together form a class of continuous basis functions, piecewise constant basis functions have inherent discontinuities or jumps.

Orthogonal functions and polynomial series have received considerable attention in dealing with various problems of dynamical systems.The main characteristic of this technique is that it reduces these problems to those of solving a system of algebraic equations, thus greatly simplifying the problem. The approach is based on converting the underlying differential equations into integral equations through integration and then approximating various signals involved in the equation by using basis vector $\Psi(t)$ and using the operational matrix of integration P to eliminate the integral equations. The matrix P is given by:

$$\int_0^t \Psi(t')dt' \simeq P\Psi(t) \qquad (1.1)$$

where $\Psi(t) = [\Psi_0(t), \Psi_1(t), \ldots, \Psi_{m-1}(t)]^T$and the matrix P can be uniquely determined on the basis of the particular orthogonal functions. The elements $\Psi_0(t), \Psi_1(t)\ \ldots, \Psi_{m-1}(t)$ are the basis functions, orthogonal on the interval$[0, 1)$.

Wavelet theory is relatively new and it has been applied in various systems.Wavelets permit the accurate representation of a variety of functions and operators. More over wavelets establish a connection with fast numerical algorithms [4]. Special attention has been given to application of the Haar wavelets [8], Legendre wavelets [15], the linear Legendre wavelets [13], the CAS wavelets [19] and the sine-cosine wavelets [16, 17].

In this paper, we introduce the Chebyshev wavelets operational matrix of integration and use it for obtaining the numerical solution of nonlinear variational problems with moving boundary conditions. The Chebyshev wavelets can be used to solve problems such as differential equations [3, 18], integral equations [2] like that of the other orthogonal functions.

The paper is organized as follows:

In section 2 we describe the formulation of the wavelets and Chebyshev wavelets. In section 3 the Chebyshev wavelets operational matrix of integration will be introduced. Section 4 is devoted to introducing variational problems with moving boundary conditions. Finally, in section5, we demonstrate the accuracy of the proposed numerical scheme by considering two numerical examples.

2. Properties of Chebyshev Wavelets

2.1. Wavelets and Chebyshev wavelets

In recent years, wavelets have found their way into many different fields of science and engineering.Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets:

$$\Psi_{a,b}(t) = |a|^{-\frac{1}{2}}\Psi\left(\frac{t-b}{a}\right) \qquad a, b \in \mathbb{R} \ , \ a \neq 0$$

Chebyshev wavelets $\Psi_{m,n}(t) = \Psi(t, m, n, k)$ have four arguments; $k = 1, 2, 3, \ldots$, $n = 1, 2, 3, \ldots, 2^k$, m is the order for Chebyshev polynomials and t is the normalized time, They are defined on the interval $[0, 1)$ by :

$$\Psi_{m,n}(t) = \begin{cases} \frac{\alpha_m 2^{\frac{k}{2}}}{\sqrt{\pi}} T_m\left(2^{k+1}t - 2n + 1\right) & \frac{n-1}{2^k} \leq t < \frac{n}{2^k} \\ 0 & o.w \end{cases} \qquad (2.1)$$

Where:

$$\alpha_m = \begin{cases} \sqrt{2} & m = 0 \\ 2 & m = 1, 2, \ldots \end{cases} \qquad (2.2)$$

Here $T_m(t)$ are the well- known Chebyshev polynomials of order m, which are orthogonal with respect to the weight function $\omega(t) = 1/\sqrt{1-t^2}$ and satisfy the following recursive formula:

$$T_0(t) = 1 \quad , \quad T_1(t) = t \qquad (2.3)$$

$$T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t) \quad , m = 1, 2, 3, \ldots$$

The set of Chebyshev wavelets are orthogonal with respect to the weight function $\omega_n(t) = \omega\left(2^{k+1}t - 2n + 1\right)$.

2.2. *Function approximation*

A function $f(t) \in L_2[0, 1)$ may be expanded as:

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\, \Psi_{n,m}(t) \qquad (2.4)$$

Where

$$c_{n,m} = \left(f(t), \Psi_{n,m}(t)\right) \qquad (2.5)$$

In equation (2.5) , (. , .) denotes the inner product with weight function $\omega_n(t)$ on the Hilbert space $L_2[0, 1)$.
If the infinite series in above equation is truncated, then equation(2.4) can be written as:

$$f(t) \simeq \sum_{n=1}^{2^k} \sum_{m=0}^{M-1} c_{n,m}\, \Psi_{n,m}(t) = C^T\Psi(t) \qquad (2.6)$$

Where $\mathbf{C}$ and $\Psi(t)$ are $2^kM \times 1$ matrices given by:

$$C = \left[c_{1,0}, c_{1,1}, \ldots, c_{1,M-1}, c_{2,0}, \ldots, c_{2,M-1}, \ldots, c_{2^k,0}, \ldots, c_{2^k,M-1}\right]^T \qquad (2.7)$$

$$\Psi(t) = \left[\Psi_{1,0}(t), \Psi_{1,1}(t), \ldots, \Psi_{1,M-1}(t), \Psi_{2,0}(t), \ldots, \Psi_{2,M-1}(t), \ldots, \Psi_{2^k,0}, \ldots, \Psi_{2^k,M-1}\right]^T \qquad (2.8)$$

3. Chebyshev wavelets operational matrix of integration

In this section, the operational matrix of integration P will be introduced. First, we find 6×6 matrix P.The six basis functions are given by:

$$\Psi_{1,0}(t) = \begin{cases} \dfrac{2}{\sqrt{\pi}} & 0 \le t < 1/2 \\ 0 & \text{o.w} \end{cases}$$

$$\Psi_{1,1}(t) = \begin{cases} \dfrac{2\sqrt{2}}{\sqrt{\pi}}(4t-1) & 0 \le t < 1/2 \\ 0 & \text{o.w} \end{cases}$$

$$\Psi_{1,2}(t) = \begin{cases} \dfrac{2\sqrt{2}}{\sqrt{\pi}}(2(4t-1)^2-1) & 0 \le t < 1/2 \\ 0 & \text{o.w} \end{cases}$$

$$\Psi_{2,0}(t) = \begin{cases} \dfrac{2}{\sqrt{\pi}} & 1/2 \le t < 1 \\ 0 & \text{o.w} \end{cases}$$

$$\Psi_{2,1}(t) = \begin{cases} \dfrac{2\sqrt{2}}{\sqrt{\pi}}(4t-3) & 1/2 \le t < 1 \\ 0 & \text{o.w} \end{cases}$$

$$\Psi_{2,2}(t) = \begin{cases} \dfrac{2\sqrt{2}}{\sqrt{\pi}}(2(4t-3)^2-1) & 1/2 \le t < 1 \\ 0 & \text{o.w} \end{cases}$$

By integrating the above six functions from 0 to t and using Eq. (2.5) we obtain:

$$\int_0^t \Psi_{1,0}(t)dt = \left(\frac{1}{4}\right)\Psi_{1,0}(t) + \left(\frac{1}{4\sqrt{2}}\right)\Psi_{1,1}(t) + \left(\frac{1}{2}\right)\Psi_{2,0}(t) \qquad (2.9)$$

$$\int_0^t \Psi_{1,1}(t)dt = \left(-\frac{1}{8\sqrt{2}}\right)\Psi_{1,0}(t) + (\frac{1}{16})\Psi_{1,2}(t) \qquad (2.10)$$

$$\int_0^t \Psi_{1,2}(t)dt = \left(-\frac{1}{6\sqrt{2}}\right)\Psi_{1,0}(t) + \left(-\frac{1}{8}\right)\Psi_{1,1}(t) + \left(\frac{-\sqrt{2}}{6}\right)\Psi_{2,0}(t) \qquad (2.11)$$

$$\int_0^t \Psi_{2,0}(t)dt = \left(\frac{1}{4}\right)\Psi_{2,0}(t) + \left(\frac{1}{4\sqrt{2}}\right)\Psi_{2,1}(t) \qquad (2.12)$$

$$\int_0^t \Psi_{2,1}(t)dt = \left(-\frac{1}{8\sqrt{2}}\right)\Psi_{2,0}(t) + \left(\frac{1}{16}\right)\Psi_{2,2}(t) \qquad (2.13)$$

$$\int_0^t \Psi_{2,2}(t)dt = \left(-\frac{1}{6\sqrt{2}}\right)\Psi_{2,0}(t) + \left(-\frac{1}{8}\right)\Psi_{2,1}(t) \qquad (2.14)$$

Thus

$$\int_0^t \Psi_6(t)dt \simeq P_{6\times 6}\Psi_6(t) \qquad (2.15)$$

Where

$$\Psi_6(t) = \left[\Psi_{1,0}(t), \Psi_{1,1}(t), \Psi_{1,2}(t), \Psi_{2,0}(t), \Psi_{2,1}(t), \Psi_{2,2}(t)\right]^T$$

By using Eqs. (2.9) - (2.15) the operational matrix of integration P is:

$$P_{6\times6} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4\sqrt{2}} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{8\sqrt{2}} & 0 & \frac{1}{16} & 0 & 0 & 0 \\ -\frac{1}{6\sqrt{2}} & -\frac{1}{8} & 0 & -\frac{\sqrt{2}}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{8\sqrt{2}} & 0 & \frac{1}{16} \\ 0 & 0 & 0 & -\frac{1}{6\sqrt{2}} & -\frac{1}{8} & 0 \end{bmatrix}$$

In Eq. (2.15) the matrix $P_{6\times6}$ can be written as:

$$P_{6\times6} = \begin{bmatrix} C_{3\times3} & S_{3\times3} \\ O_{3\times3} & C_{3\times3} \end{bmatrix}$$

In general, we have:

$$\int_0^t \Psi(t)ds \simeq P\,\Psi(t)$$

Where $\Psi(t)$ has been given in equation (2.8) and P is a $(2^k M) \times (2^k M)$ matrix given by:

$$P = \begin{bmatrix} C & S & S & \dots & S \\ O & C & S & \dots & S \\ O & O & C & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & S \\ O & O & O & \dots & C \end{bmatrix}$$

Where S, C are $M \times M$ matrices as follows:

$$S=\frac{\sqrt{2}}{2^k}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ -\frac{1}{3} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ -\frac{1}{15} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{M(M-2)} & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$C=$$

$$\frac{1}{2^k}\begin{bmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & \dots & 0 & 0 & 0 \\ -\frac{1}{4\sqrt{2}} & 0 & \frac{1}{8} & 0 & \dots & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{2}} & -\frac{1}{4} & 0 & \frac{1}{12} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -\frac{1}{2\sqrt{2}(M-1)(M-3)} & 0 & 0 & 0 & \dots & -\frac{1}{4(M-3)} & 0 & -\frac{1}{4(M-1)} \\ -\frac{1}{2\sqrt{2}M(M-2)} & 0 & 0 & 0 & \dots & 0 & -\frac{1}{4(M-2)} & 0 \end{bmatrix}$$

4. Variational problems with moving boundary conditions

In the large number of problems arising in analysis, mechanics, geometry, and so forth, it is necessary to determine the maximal and minimal of a certain functional. Because of the important role of this subject in sciences and engineering, this kind of problems have received considerable attention, such problems are called variational problems.
If the initial point $x(t_0)$ is costant and t_0 is free, the simplest form of a vriational problem is finding extremum of the functional:

$$J\big(x(t)\big)=\int_{t_0}^{t_1} F(\,t\,,x(t),\dot{x}(t)\,)\,dt \qquad (4.1)$$

Subject to

$$x(t_1) = \beta \qquad (4.2)$$

The necessary conditions for $x(t)$ to extremize $J(x)$ are:

Euler-Lagrange equation: $\frac{\partial F}{\partial x} - \frac{d}{dt}\frac{\partial F}{\partial \dot{x}} = 0$

Transversality condition : $F(t_0) - \dot{x}(t_0)\frac{\partial F}{\partial \dot{x}}(t_0) = 0 \quad (4.3)$

If the initial time t_0 is constant and $x(t_0)$ is free then the necessary conditions for solving the above variational problem are:

Euler-Lagrange equation : $\frac{\partial F}{\partial x} - \frac{d}{dt}\frac{\partial F}{\partial \dot{x}} = 0$

Transversality condition : $\frac{\partial F}{\partial \dot{x}}(t_0) = 0 \qquad (4.4)$

Similarly, we can consider the above formulae in the case that one of the values of t_1 or $x(t_1)$ is free.

5. Numerical solution

Example(1). Find the extremal of the following functional:

$$J = \int_0^1 \dot{x}^2(t) + t\dot{x}(t)dt \qquad (5.1)$$

With boundary conditions:

$$x(0) = 0 \qquad (5.2)$$

$$x(1) \quad \text{unspecified} \qquad (5.3)$$

We solve this problem by using Chebyshev wavelets, First we assume:

$$\dot{x}(t) \simeq C^T\Psi(t) = \Psi(t)^T C \qquad (5.4)$$

We can also express variable t in Eq. (5.1) in terms of Chebyshev wavelets as:

$$t \simeq d^T\Psi(t) \qquad (5.5)$$

Substituting Eqs. (5.4) and (5.5) into Eq. (5.1), we have :

$$J \simeq \int_0^1 [C^T\Psi(t)\,\Psi(t)^T C + d^T\Psi(t)\Psi(t)^T C]\,dt \qquad (5.6)$$

By considering:

$$\int_0^1 \Psi(t)\Psi(t)^T\,dt = R \qquad (5.7)$$

Eq. (5.6) is simplified as:

$$J \simeq C^T RC + d^T RC \qquad (5.8)$$

By using Eqs. (5.2) and (5.4) the variable $x(t)$ can be expressed as:

$$x(t) = \int_0^t \dot{x}(\tau)\,d\tau + x(0) \simeq C^T \int_0^t \Psi(\tau)\,d\tau + x(0) \simeq C^T P\Psi(t) \qquad (5.9)$$

According to transversality condition we have:

$$\left.\frac{\partial F}{\partial \dot{x}}\right|_{t=1} = 0 \quad \Longrightarrow \quad \dot{x}(1) = -1/2$$

By (5.4):

$$\dot{x}(1) \simeq C^T\Psi(1) = -1/2$$

By doing so, the variational problem is reduced to extremization of the following function:

$$J = C^T RC + d^T RC$$

Subject to:

$$C^T\Psi(1) = -\frac{1}{2}$$

Let

$$J^* = J + \lambda\left(C^T\Psi(1) + \frac{1}{2}\right) \qquad (5.10)$$

Where λ is unknown Lagrange multiplier, then the necessary conditions for extremum are:

$$\frac{\partial J^*}{\partial C^T} = 2RC + R^T d = 0$$
$$\frac{\partial J^*}{\partial \lambda} = C^T\Psi(1) + \frac{1}{2} = 0 \qquad (5.11)$$

For $M = 3$ and $k = 1$ we obtain:

$$d = \begin{bmatrix} \frac{\sqrt{\pi}}{8} & \frac{\sqrt{2\pi}}{16} & 0 & \frac{3\sqrt{\pi}}{8} & \frac{\sqrt{2\pi}}{16} & 0 \end{bmatrix}^T$$

And

$$R = \begin{bmatrix} \frac{2}{\pi} & 0 & -\frac{2\sqrt{2}}{3\pi} & 0 & 0 & 0 \\ 0 & \frac{4}{3\pi} & 0 & 0 & 0 & 0 \\ -\frac{2\sqrt{2}}{3\pi} & 0 & \frac{2}{\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\pi} & 0 & -\frac{2\sqrt{2}}{3\pi} \\ 0 & 0 & 0 & 0 & \frac{4}{3\pi} & 0 \\ 0 & 0 & 0 & -\frac{2\sqrt{2}}{3\pi} & 0 & \frac{2}{\pi} \end{bmatrix}$$

Thus

$$C = [-0.1108 \;\; -0.0783 \;\; 0 \;\; -0.3323 \;\; -0.0783 \;\; 0]^T$$

Analytic solution via Euler's equation is:

$$\dot{x}(t) = -t/2$$
$$x(t) = -t^2/4$$

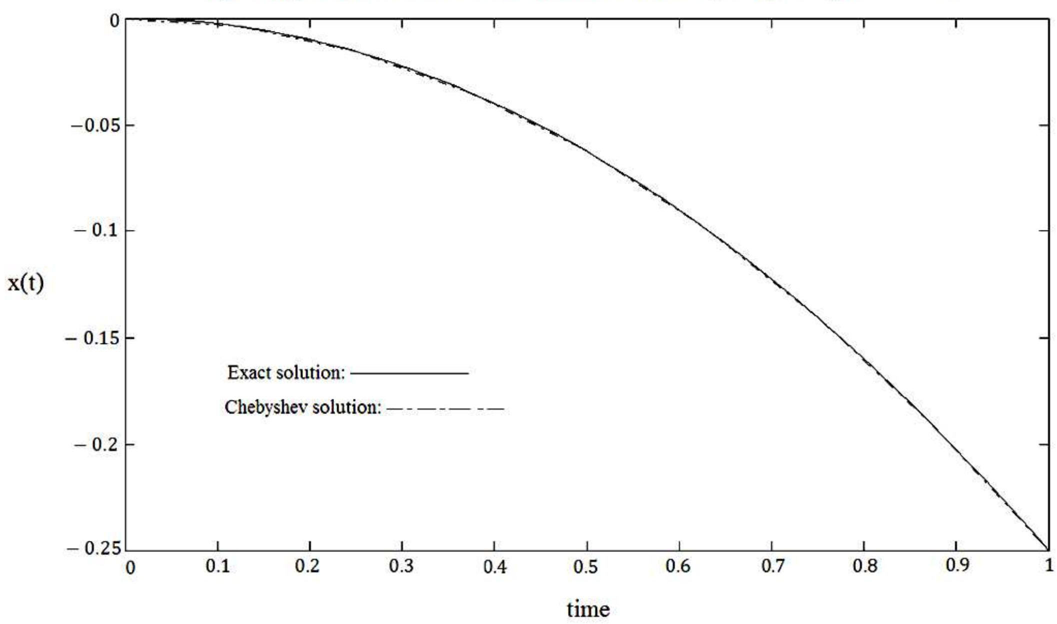
Table 1 and fig.1 show numerical results of the above example with $M = 3$ and $k = 1$.

Also, the solution obtained by sine-cosine wavelets (by using 28 basis functions) has been shown in table1.

Table Numerical results of example 1

Time	sine-cosine wavelets	Chebyshev wavelets	Exact solution
0	-0.0078	0	0
0.125	-0.0043	-0.0039	-0.0039
0.250	-0.0391	-0.0156	-0.0156
0.375	-0.0355	-0.0352	-0.0352
0.500	-0.1016	-0.0625	-0.0625
0.625	-0.0980	-0.0977	-0.0977
0.750	-0.1964	-0.1406	-0.1406
0.875	-0.1929	-0.1914	-0.1914
1	-0.2522	-0.2500	-0.2500

Fig1. comparison exact solution and numerical solution by using Chebyshev wavelets



Example(2).Suppose we want to find the following functional extremal:

$$J = \int_0^1 \left[\frac{\ddot{x}^2(t)}{2} + (4-4t)\dot{x}(t)\right] dt \qquad (5.12)$$

Subject to

$$x(0) = 0$$

$$\dot{x}(0) = 0 \qquad (5.13)$$

$$x(1)\,,\dot{x}(1)\ \text{unspecified}$$

From Euler-Lagrange equation we have:

$$\frac{\partial F}{\partial \dot{x}} - \frac{d}{dt}\frac{\partial F}{\partial \ddot{x}}\bigg|_{t=1} = 0 \quad \Longrightarrow \quad (4-4t) - \dddot{x}(t)\big|_{t=1} = 0 \quad \Longrightarrow \quad \dddot{x}(1) = 0 \quad (5.14)$$

Also, transversality condition implies:

$$\frac{\partial F}{\partial \ddot{x}}\bigg|_{t=1} = 0 \qquad \Longrightarrow \ddot{x}(1) = 0 \qquad (5.15)$$

For solving this problem by the Chebyshev wavelets, let:

$$\dddot{x}(t) \simeq C^T\Psi(t) \qquad (5.16)$$

By integrating $\dddot{x}(\tau)$ from 0 to t and using Eqs (5.16) and (1.1) we get:

$$\ddot{x}(t) = \int_0^t \dddot{x}(\tau)d\tau + \ddot{x}(0) \simeq C^T \int_0^t \Psi(\tau)d\tau + \ddot{x}(0) \simeq C^T P\Psi(t) + \ddot{x}(0) \qquad (5.17)$$

From (5.14) and (5.16) we have:

$$\dddot{x}(1) \simeq C^T\Psi(1) \simeq 0$$

In view of Eqs (5.15) and (5.17)

$$\ddot{x}(1) \simeq C^T \int_0^1 \Psi(\tau)d\tau + \ddot{x}(0) \simeq 0$$

Let, $V = \int_0^1 \Psi(t')dt'$, we have:

$$\ddot{x}(0) \simeq -C^T V \simeq C^T Q\,\Psi(t) \qquad (5.18)$$

That the quadratic matrix Q must be found.

Thus, Eq. (5.17) can be written as:

$$\ddot{x}(t) \simeq C^T(P+Q)\Psi(t) \qquad (5.19)$$

Also, in view of boundary conditions and Eq (1.1) we have:

$$\dot{x}(t) = \int_0^t \ddot{x}(t')dt' + \dot{x}(0) \simeq C^T(P+Q)\int_0^t \Psi(t')dt' \simeq C^T(P+Q)P\Psi(t) \qquad (5.20)$$

Let,

$$4 - 4t \simeq d^T\Psi(t)$$

Now, by using Eqs. (5.19) and (5.20) the cost function J, may be rewritten as:

$$J(x) \simeq \int_0^1 [\frac{1}{2}(C^T(P+Q)\Psi(t)\Psi^T(t)\left(P^T+Q^T\right)C) + d^T\,\Psi(t)\Psi^T(t)P^T\left(P^T+Q^T\right)C]\,dt$$

Thus,(5.7) follows that:

$$J(x) \simeq \frac{1}{2}C^T(P+Q)R(P^T+Q^T)C + d^T R P^T(P^T+Q^T)C$$

Consider,

$$J^*(C,\lambda) = J + \lambda C^T\Psi(1)$$

Where λ is unknown Lagrange multiplier, Then the necessary conditions for extremum are:

$$\frac{\partial J^*}{\partial C^T} = (P+Q)R(P^T+Q^T)C + (P+Q)PR^T d + \lambda\Psi(1)$$

$$\frac{\partial J^*}{\partial \lambda} = C^T\Psi(1)$$

For $M = 3$ and $k = 1$ we obtain:

$$\mathrm{d} = [2.6587\ \ -0.6267\ \ 0\ \ 0.8862\ \ -0.6267\ \ 0]^{\mathrm{T}}$$

$$Q = \begin{bmatrix} -0.5 & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2357 & 0 & 0 & 0.2357 & 0 & 0 \\ -0.5 & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2357 & 0 & 0 & 0.2357 & 0 & 0 \end{bmatrix}$$

$$\mathrm{C} = [2.6590\ \ -0.3581\ \ 4.8297\mathrm{e}-004\ \ 0.7795\ \ 0.4591\ \ -0.0921]^{\mathrm{T}}$$

Analytic solution via Euler's equation is:

$$\ddot{\mathrm{x}}(\mathrm{t}) = -2\mathrm{t}^2 + 4\mathrm{t} - 2$$

$$\dot{\mathrm{x}}(\mathrm{t}) = -\left(\frac{2}{3}\right)\mathrm{t}^3 + 2\mathrm{t}^2 - 2t$$

$$\mathrm{x}(\mathrm{t}) = -\frac{\mathrm{t}^4}{6} + \left(\frac{2}{3}\right)\mathrm{t}^3 - \mathrm{t}^2$$

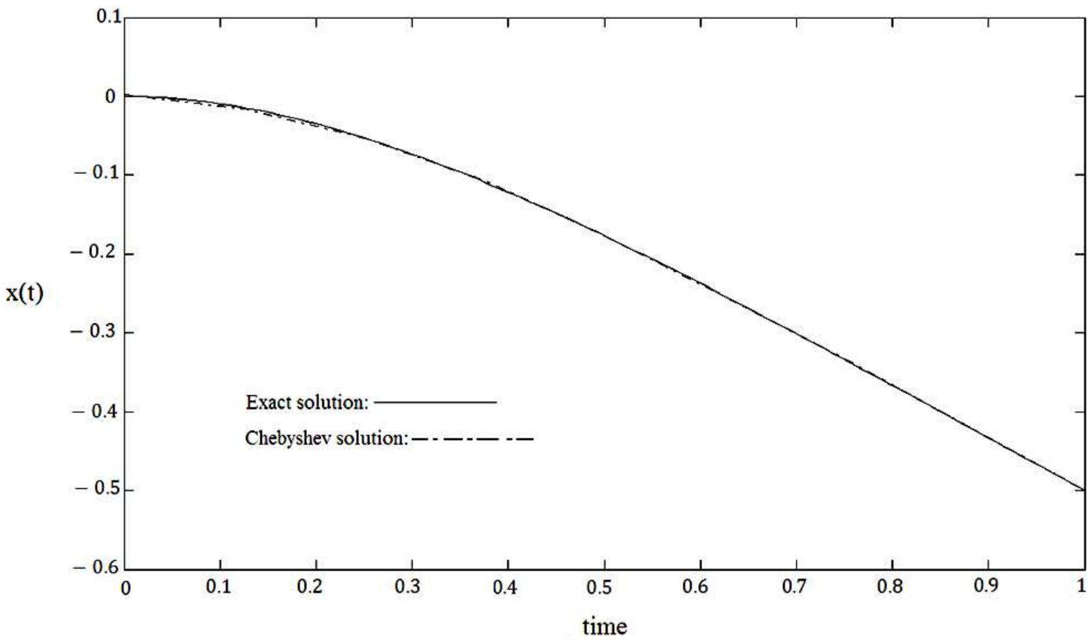
Table 2 shows numerical results of the above example with both Chebyshev(with $\mathrm{M} = 3, \mathrm{k} = 1$) and sine-cosine (by using 20 basis functions) wavelets methods.

Also, the comparison between results for $\mathrm{x}(\mathrm{t})$ by applying presented method with $\mathrm{M} = 3$ and $\mathrm{k} = 1$ and the exact solution of $\mathrm{x}(\mathrm{t})$ is shown in fig. 2.

Table Numerical results of example 2

Time	sine-cosine wavelets	Chebyshev wavelets	Exact solution
0	*-0.0253*	0.002	0
0.125	-0.0155	-0.0161	-0.0144
0.250	-0.0119	*-0.0523*	-0.0527
0.375	-0.1069	*-0.1066*	-0.1088
0.500	-0.2513	-0.1765	-0.1771
0.625	-0.2494	-0.2537	-0.2533
0.750	*-0.4119*	*-0.3335*	-0.3340
0.875	-0.4116	-0.4158	-0.4167
1	-0.4946	-0.5001	-0.5000

Fig.2 .comparison exact solution and numerical solution by using Chebyshev wavelets



In [4] two above examples have been solved by using eight basis functions, whereas, we have obtained similar satisfactory results by applying six basis functions.

6. Conclusion

In this paper we have developed an accurate method for solving nonlinear variational problems. The Chebyshev wavelet 6×6 operational matrix of integration has been derived in details directly, and then a general formulation for the above matrix has been given. The Chebyshev wavelet operational matrix of integration is used to reduce the variational problems to solving a system of linear algebraic equations.

Moreover, only a small number of Chebyshev wavelets are needed to obtain satisfactory results. The above examples support this claim.

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